Fermilab UPC 104

SOME CALCULATIONS ABOUT THE HEATERS USED IN THE EDS DIPOLES

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Introduction

The general protection scheme of the Energy Saver makes a large use of heaters to insure rapid and even dissipation of the stored energy in case of quench.[1] A lot of measurements were done a few months ago to study the best way to induce and propagate the quench, the variable parameters being the kind of heater and discharge circuit used.[2] These experimental results led to some choice.

The physical problem can be divided in two very different parts: (a) The energy propagation from the heater to the conductor to induce the quench; (b) The quench propagation inside the coil, assuming no dump resistance.

In this memo, a quite simple theory has been developed for the first point. The comparison with experimental results is good enough to draw more general results. The second point is also indicated, but in a very different manner, using an available program.

Both unfortunately and fortunately this study does not suggest big changes for a large improvement. Only small changes are suggested, mostly as money savers.

I. The Theoretical Model

I-1 Some recalls about the heaters. Three kinds of heaters have been used up to now (Fig. 1):

(a) Standard heater: put on top of the outer coil. Will be referred to later as H1;

(b) Ground wrap heater: same as H1 but with more Mylar insulation (H2).

(c) Extended heater: put on the outer side of the outer coil: H3. This is the kind of heater which is now put on the TA dipoles.

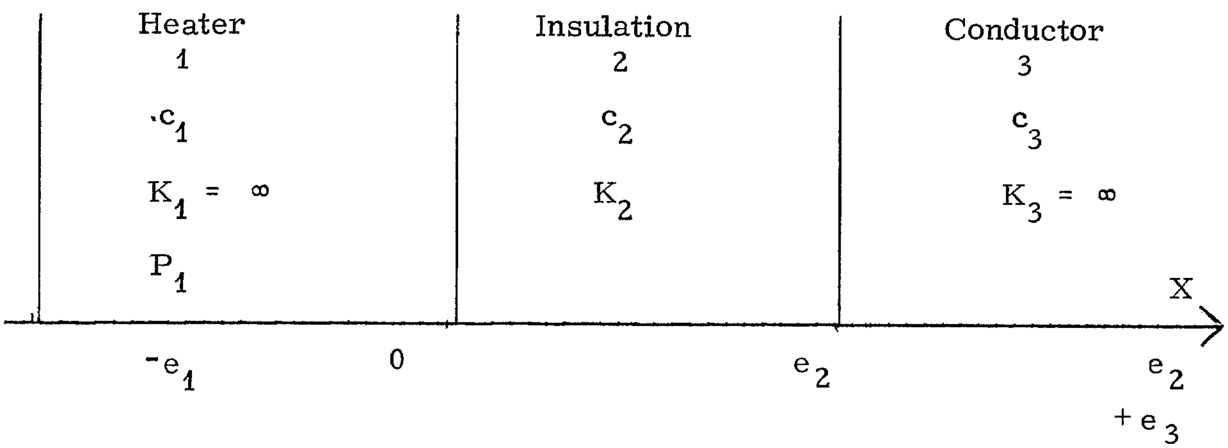
A 20-ft long stainless-steel ribbon is used to make the heaters. The two ribbons on one coil are connected in series to make one heater. So, there are two heaters per dipole. The electrical insulation is done either with Kapton or with Mylar.

The firing circuit uses a capacitance maintained under a high voltage. Most of the time only one heater per dipole is fired.

It is important to remember that channels are made around the conductor using a glass tape, spiral wrapped with a gap. These channels are filled with helium when the heater is fired.

<u>I-2 The model used.</u> The calculations done here have some similarities with previous ones.[3] The principal characteristics of the model used are: (a) One dimension composite model; (b) The Laplace transform is used to study the time dependence.

The materials are supposed with constant properties in space and time. The same model will be used for the 3 kinds of heaters. Only the dimensions and the material properties are different.



The following assumptions are made about the components:

Heater: thickness e_1

specific heat c_1

thermal conductivity $K_1 = \infty$

heat generated by unit length: $p_1 = p_0 \, e^{-\frac{2t}{\tau}}$

with $p_0 = \dfrac{V^2}{Re_1}$ $\tau = Rc$

where R, C and V are the resistance of the heater (at 4K), the capacitance and the voltage of the firing circuit.

insulation: e_2, c_2, K_2

conductor: e_3, c_3, $K_3 = \infty$

The initial temperature is t_0 everywhere

σ denotes the increase of temperature above t_0.

ϕ is used for thermal flux.

The indices 1, 2, 3 refer to each slab.

Boundary conditions:

No flux flow on one side of the heater:

$$\phi_1 (-e_1, t) = 0 \tag{1}$$

Between slab 1 and 2:

$$\phi_1 (0, t) = \phi_2 (0, t) \tag{2}$$

$$\sigma_1 (0, t) = \sigma_2 (0, t) \tag{3}$$

At the entrance of the conductor:

$$\sigma_2 (e_2, t) = 0 \tag{4}$$

(this condition is justified by the fact that the increase of temperature in the conductor to reach its nominal temperature is much lower than the increase of temperature anywhere else).

I-3 Resolution of the equations. We need to calculate the flux entering the conductor: $\phi_2\ (e_2,\ t)$. Writing the equation in the first slab:

$$\frac{\partial \phi_1(\kappa, t)}{\partial \kappa} = - c_1 \frac{\partial \sigma_1(\kappa, t)}{\partial t} + \rho_1 \tag{5}$$

or in Laplace transform:

$$\frac{d\,\overline{\phi}_1(\kappa, p)}{d\kappa} = - c_1 p\,\overline{\sigma}_1\ (\kappa, p) + \frac{\rho_0}{p + \frac{2}{\tau}}\ . \tag{5'}$$

Integrating this equation and using (1):

$$\overline{\phi}_1(\kappa,\ p) = \left[- c_1\, p\ \overline{\sigma}_1(\kappa, p) + \frac{\rho_0}{p + \frac{2}{\tau}} \right] (\kappa + \rho_1) \tag{6}$$

Similarly, in slab 2:

$$c_2 \frac{\partial \sigma_2\ (\kappa, t)}{\partial t} = \kappa_2 \frac{\partial^2 \sigma_2\ (\kappa, \tau)}{\partial \kappa^2} \tag{7}$$

or:

$$c_2\, p\, \overline{\sigma}_2\ (\kappa,\ p) = \kappa_2 \frac{d^2 \sigma_2\ (\kappa, p)}{d\kappa^2} \tag{7'}$$

Integrating:

$$\overline{\sigma}_2\ (\kappa,\ p) = A_2\ \ \mathrm{sh}(\sqrt{u}\ \kappa) + B_2\ \ \mathrm{ch}\,(\sqrt{u}\ \kappa) \tag{8}$$

with

$$u = \frac{c_2\, p}{\kappa_2} \tag{9}$$

So:

$$\phi_2\ (\kappa,\ p) = - \kappa_2 \sqrt{u}\ \left[A_2\ \ \mathrm{ch}\,(\sqrt{u}\kappa) + B_2\ \ \mathrm{sh}\,(\sqrt{u}\,\kappa)\right] \tag{10}$$

From (4) and (8):

$$A_2 = - B_2 \frac{ch(\sqrt{u}\; e_2)}{sh(\sqrt{u}\; e_2)} \tag{11}$$

From (3) and (8)

$$\bar{\sigma}_1 (0, \rho) = B_2 \tag{12}$$

Using (6) and (10), (2) can be written as:

$$\left[- c_1 \; \rho \, \bar{\sigma}_1 \; (0, \; \rho) + \frac{\rho_0}{\rho + \frac{2}{\tau}} \right] e_1 = - \kappa_2 \; \sqrt{u} \; A_2 \tag{13}$$

From (11), (12) and (13):

$$B_2 = \frac{\rho_0 \, e_1}{\left(\rho + \frac{2}{\tau}\right) \left[c_1 \; \rho \; e_1 + \kappa_2 \sqrt{u} \, \frac{ch(\sqrt{u} e_2)}{sh(\sqrt{u} e_2)} \right]} \tag{14}$$

and finally from (10), (11) and (14):

$$\bar{\phi}_2 (e_2, \; \rho) = \frac{\rho_0 \; e_1}{\rho + \frac{2}{\tau}} \; \frac{1}{ch\left(\sqrt{\frac{c_2 \, \rho}{\kappa_2}} \; e_2\right) + c_1 \; e_1 \sqrt{\frac{\rho}{\kappa_2 c_2}} \; sh\left(\sqrt{\frac{c_2 \rho}{\kappa_2}} \; e_2\right)} \tag{15}$$

There is no simple general solution to this equation. To solve it, we assume that the time we are interested in is large enough (this means ρ small) to develop the trigonometric functions in series. We have verified that as the series is quickly convergent, one can stop the development at the first order. In that case:

$$\bar{\phi}_2 \; (e_2, \; \rho) = \frac{\rho_0 \; e_1}{\rho + \frac{2}{\tau}} \qquad \frac{1}{1 + \omega \rho} \tag{16}$$

with

$$\omega = \frac{e_2}{\kappa_2} \left(\frac{c_2 \; e_2}{2} + c_1 \; e_1 \right) \tag{17}$$

So

$$\phi_2\ (e_2, t) = \frac{\rho_0\ e_1\ \tau}{2\ \omega\ -\tau} \left(e^{-\frac{t}{\omega}} - e^{-\frac{2t}{\tau}}\right) \tag{18}$$

As the thermal conductivity of the conductor is supposed infinite, the energy in the conductor is:

$$W_3\ (t) = \int \phi_2\ (e_2, t)\ d\ x\,dt \tag{19}$$

So

$$W_3\ (t) = \frac{W_H}{2\omega\ -\ \tau} \left[2\omega\ (1-e^{-\frac{\tau}{\omega}}) - \tau\ (1 - e^{-\frac{2t}{\tau}})\right] \tag{19'}$$

introducing the total energy put in the heater:

$$W_H = \frac{\rho_0\ e_1\ \tau}{2} = \frac{1}{2}\ c\ V^2 \tag{20}$$

the heater time constant

$$\tau = Rc \tag{20'}$$

and the heat propagation time constant:

$$\omega = \frac{e_2}{\kappa_2} \left(\frac{c_2\ e_2}{2} + c_1\ e_1\right) \tag{20''}$$

II. Comparison With The Experimental Results

II-1 Critical temperature of the conductor facing the heater. We assume that the ratio of the magnetic field on the conductor facing the heater and the central field is:[4]

0.8 for H_1 and H_2

0.6 for H_3 (mean value on the outer part of the coil).

Also, remember that the experiments were done without the iron. So the following values are taken:

I (A)	2000	3000	4300
B on cond (kG)			
H_1, H_2	13	20	27
H_3	10	15	21
$J_{sc}(A/m^2)$	7 E 8	1.0 E 9	1.4 E 9
T_c (k)			
H_1, H_2	8.0	7.2	6.2
H_3	8.3	7.6	6.7

The critical temperature as a function of the magnetic field and the current density was determined using Ref. 5.

II-2 Energy to bring the conductor to its normal state. The following assumptions are made:

Initial temperature: $T_0 = 4.2$ K

Conductor specific heat and enthalpy:

$$c_{cond} = \frac{1.8\ c_{cu} + 1.0\ c_{sc}}{2.8}$$

with the following laws for copper, superconductor and insulation ($T < 10$ K)

$$c_{cu} = 90\ T + 6.3\ T^3 \quad J\ |\ m^3|\ K$$

$$c_{sc} = 40\ T^3 \quad J\ |\ m^3|\ K$$

$$c_{ins} = 200\ T^{2-1} \quad J\ |\ m^3|\ K$$

Volume of conductor to be quenched:

H_1 and H_2: The first conductor facing the heater, on the heater length:

$V_{c1,2} = 0.308'' \times 0.05'' \times 20' \times 12 \times 2 = 7.39 \text{ in}^3$

$= 120 \text{ cm}^3$

H_3: 10 conductors and their insulation are facing the heater:

Volume of conductor $V_{c3} = 1200 \text{ cm}^3$

Volume of insulation $V_{i3} = 20 \text{ in}^3 = 300 \text{ cm}^3$

Combining all these elements, one finally finds the energy to bring the conductor to its normal state.

I (A)	2000	3000	4300
W_{cond} (J)			
H_1, H_2	2.5	1.7	0.7
H_3	41	29	16

II-3 Energy to vaporize the helium between the heater and the conductor. For H_2 and H_3, there is helium between the heater and the conductor. This helium will be heated during the process.

Volume to be considered: We assume first, that because of the pressure process, the initial channel height is divided by a factor of 2.

H_2: Channels on one side of the conductor.

$V_2 = 0.5 \times .308'' \times 7\ 10^{-3''} \times 20' \times 12 \times 2 \times .33 = 0.17 \text{ in}^3 = 2.8 \text{ cm}^3$

H_3: Channels on the side of the conductors facing the heater and channels between the 10 conductors:

$V_3 = 0.5 \times 7\ 10^{-3''} \times .33 \times 20' \times 12 \times 2 \times (.75'' + 20 \times .308'')$

$= 3.8 \text{ in}^3 = 62 \text{ cm}^3$

Enthalpy of the helium at 2 bar: We assume that the main part of the energy to be considered is to bring to helium to its vaporization point:

$$H_{4.2\,K}^{5.047\,K} = 0.196\,J/cm^3$$

So the energy to heat the helium is:

$$W_{Hel\,2} = 0.5\ J$$

$$W_{Hel\,3} = 12\ J$$

<u>II-4 Total energy taken into account.</u> Summing up the different energies to take into account, one finds for the total energy to be considered:

I (A)	2000	3000	4300
$W_{T\,H1}$(J)	2.5	1.7	0.7
$W_{T\,H2}$(J)	3.0	2.2	1.2
$W_{T\,H3}$(J)	53	41	28

<u>II-5 Calculation of ω.</u> This is the only unknown in the formula (19').

The following assumptions have been done for its calculation:

The experimental time t_1 when the quench is induced is known.

The energy in the heater at this time is calculated:

$$W(t_1) = W_H \left(1 - e^{-\frac{2\,t_1}{\tau}}\right)$$

with $$W_H = \frac{1}{2}\,cv^2$$

This energy is assumed to bring adiabatically the heater at the temperature T_H.

The temperature of the insulation is assumed to be:

$$T_{ins} = \frac{T_H}{2}$$

c_1 is calculated for T_H, c_2 and K_2 for T_{ins}.

To take into account the actual geometry, we assume that there is a factor 0.5 in the energy transmission from the heater to the conductor.

II-6 Comparison with the experimental results. The tables I to IV give for different magnets the values of:

The experimental conditions: I, C, V, τ

The value of ω (calculated) and t_1

(experimental time when the quench starts).

The value of W_T cal, total energy needed to quench the conductor, as calculated in II-4.

The value of $W_3(t_1)$ as calculated using (19'). If the model is correct, this value must agree with W_T cal.

MAGNET # PCA 135 HEATER # 1 $R_H = 10.3\ \Omega$

I (A)	C (μF)	V (v)	W_H (J)	τ (msec)	ω (msec)	$t_1$exp (msec)	$W_T$cal (J)	$W_3(t_1)$ (J)
2000	653	300	29	7	120	60	2.5	5.5
		350	40		160	50		5.0
	3200	140	31	33	120	85		6.7
		160	41		160	70		5.8
		180	52		160	65		6.8
		200	64		160	60		7.6
		300	144		240	45		8.2
		325	167		240	40		8.2
		350	196		240	40		9.6
	9600	107	55	100		140		11.5
3000	653	250	20	7	120	55	1.7	3.5
		350	40		160	40		4.1
		400	52		160	35		4.6
	3200	128	26	33	120	75		5.0
		192	59		160	65		7.7
		300	144		240	35		5.8
		344	190		240	35		7.6
	9600	108	56	100	160	70		4.8
		204	200		240	50		7.2
		303	440		270	40		9.7
4300	653	300	29	7	120	35	0.7	3.3
		400	52		160	30		4.0
	3200	200	64	33	160	35		3.8
		300	144		240	30		4.6
	9600	100	48	100	160	50		2.5
		200	192		240	35		3.6
		300	432		270	25		4.3

Table I

MAGNET # PCA 141 HEATER # 2 R_H = 10.5 Ω

I (A)	C (μF)	V (v)	W_H (J)	τ (msec)	ω (msec)	$t_1$exp (msec)	$W_T$cal (J)	$W_3(t_1)$ (J)
2000	653	300	29	7	450	105	3.0	2.9
		350	40		575	95		3.0
	3200	140	31	34	450	185		4.9
		200	64		575	110		4.8
		300	144		840	55		3.3
	9600	107	55	100	575	120		3.3
		250	300		1000	80		5.7
		300	432		1000	60		5.2
3000	653	250	20	7	450	80	2.2	1.6
		350	40		575	50		1.6
		400	52		575	45		1.8
	3200	128	26	34	450	110		2.4
		192	59		575	56		2.4
		300	144		840	55		3.3
		344	190		840	45		3.2
	9600	204	200	100	840	45		1.8
		303	440		1000	35		2.2
4300	653	300	29	7	450	45	1.2	1.3
		400	52		575	45		1.8
	3200	900	64	34	575	40		1.3
		300	144		840	30		1.4
	9600	100	48	100	575	65		1.2
		200	192		840	35		1.2
		300	432		1000	30		1.7

Table II

MAGNET # PCA 142 HEATER # 2 R_H = 10.1 Ω

I (A)	C (μF)	V (v)	W_H (J)	τ (msec)	ω (msec)	$t_1$exp (msec)	$W_T$cal (J)	$W_3(t_1)$ (J)
2000	653	300	29	7	450	110	3.0	3.0
		350	40		575	65		2.0
	3200	140	31	32	450	165		4.3
		160	41		575	80		2.1
		180	52		575	70		2.3
		200	64		575	70		2.8
		300	144		840	45		2.4
		350	196		840	40		2.9
	9600	107	55	97	575	95		2.4
		250	300		1000	50		2.7
		300	432		1000	40		2.6
3000	653	350	40	7	575	50	2.2	1.6
	3200	130	27	32	450	90		2.0
		190	58		575	80		3.0
		300	144		840	35		1.7
		345	100		840	40		2.9
	9600	108	56	97	575	75		1.7
		207	200		840	40		1.4
		303	400		1000	45		3.2

Table III

MAGNET # PCA 152 HEATER # 3 R_H = 17.7 Ω

I (A)	C (μF)	V (v)	W_H (J)	τ (msec)	ω (msec)	$t_1$exp (msec)	$W_T$cal (J)	$W_3(t_1)$ (J)
3000	9600	300	432	170	200	100	41	38
4300	1000	1000	500	18	210	30	28	24
	9600	300	432	170	200	70		21
	9600x2	300x2	432x2	85	210	70		32

Table IV

The following comments can be done - from the experimental results first: even for two magnets supposed similar (PCA 141 and 142), the time to induce the quench in similar conditions is different.

The difference in W for H1 and H2 is mostly explained by the Mylar which has a much smaller thermal conductivity than Kapton, according to Ref. 6.

PCA 135:

W_3 (t_1) is always larger than the calculated energy needed. We can think that this heater was not very well thermally insulated.

PCA 141 and 142:

The results are in better agreement, most of them being in a window of about ± 50 %.

PCA 152:

There are much less experimental results, they are in quite good agreement with the model (~ ± 25%).

In conclusion, it is very hard to take into account all the physical phenomena with their correct values (presence of helium, heat loss...). However, the quite simple model made gives quite consistent results for very different experimental conditions (I, W_H, τ); in two cases out of three, the accuracy is better than 50%, which does not seem so bad for this kind of problem. Se we can later use it as a guide.

III. Applications of the Theory.

The question is: Is there an optimum in the heater configuration to decrease the number of MIITS before the quench is induced ? We now look at different points (without giving all the calculation details).

III-1 Heater constitution. Stainless-steel is OK as a material. There is a small interest to increase the thickness of the heater (R decreases but ω increases). The extended heater needs much more energy, has a larger time constant and so it is not the most interesting from this point of view. It's only interest can be to propagate the quench quicker.

III-2 Energy to put in the heater. As a general fact, it is interesting to increase the energy in the heater, but (Fig. 2);

It is much more interesting to do that by increasing the voltage.

With a constant voltage, there is an upper value after which the gain is small.

For the extended heaters now used, small variations around the firing circuit (9600 μF, 300V) give the following change in MIITS:

C/V	300 V	350 V
9600 μF	Ref.	-0.2
6400 μF	+ 0.2	0
3200 μF	+ 0.6	+ 0.4

So, it is possible to reduce the number of capacitances in parallel to 2, almost without change for the MIITS number.

IV. Quench Propagation

Once the quench has been initiated by the heater, it propagates inside the coil.

The "quench" program [7] has been used to study this part. Its main assumptions are:

The coil is developed as a parallel piped.

The quench is induced in one point.

The quench velocity along the conductor is proportional to some power of the current (here: $V \propto I$). Its initial value can be either calculated or imposed.

The quench velocities in each direction are proportional to $\kappa^{\frac{1}{2}}$, κ being the average thermal conductivity in the direction.

Average values for C and κ are taken into account and are temperature dependent.

There is no heat exchange between the coil and the outside.

To meet these requirements, the following assumptions were done:

Coil height: Average value between the inner and the outer coil.

Coil length: Upper and lower coil in series.

Quench velocities: the value along the conductor is calculated by the program (19m/sec). For the transversal values, the different materials (copper, superconductor, insulation, helium) are taken into account and the following ratios were calculated:

$$H_1, H_2 \begin{cases} \alpha = \dfrac{\kappa_x}{\kappa_z} = 1.4 \ 10^{-2} \\ \epsilon = \dfrac{\kappa_y}{\kappa_z} = 2.7 \ 10^{-3} \end{cases}$$

For H3, ϵ is taken equal to 1 to simulate the fact that the whole layer is quenched when the program starts:

$$H3 \quad \begin{cases} \alpha = 1.4 \ 10^{-2} \\ \epsilon = 1 \end{cases}$$

The results obtained with the program are compared to experimental ones on Fig. 3. The agreement is quite good for H3, medium for H1. The curves show the interest of the extended heater for that point of view. The only way to increase the current decay in that case would be to increase the transversal conductivities, but this would lead to important changes and so is out of question.

Conclusions

The reason for this study was a better understanding of experimental results. A theory which can explain then with a reasonable accuracy has been carried out and applied for very different experimental conditions. The important parameters have been drawn: energy to put in the heater, time constant, thermal propagation.

The only improvement which can be suggested is to reduce the number of parallel capacitances from 3 to 2. This will not change the results but will save money. Perhaps, a better thermal insulation of the heater can be done (Kapton being worst than Mylar from that point of view).

I would like to thank K. Koepke and A. Tollestrup for valuable discusions and suggestions about this problem.

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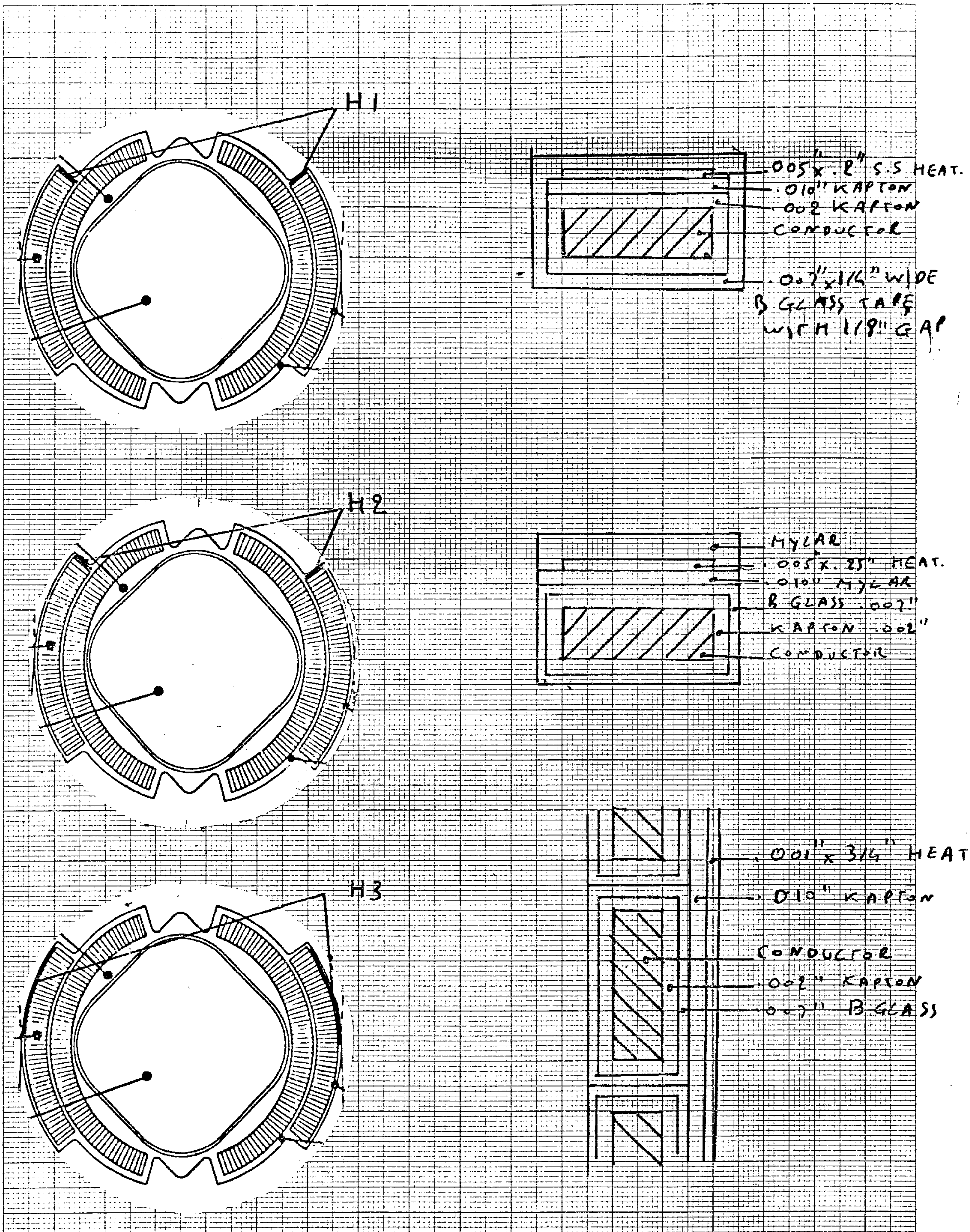

Fig 1: the different Kinds of heaters

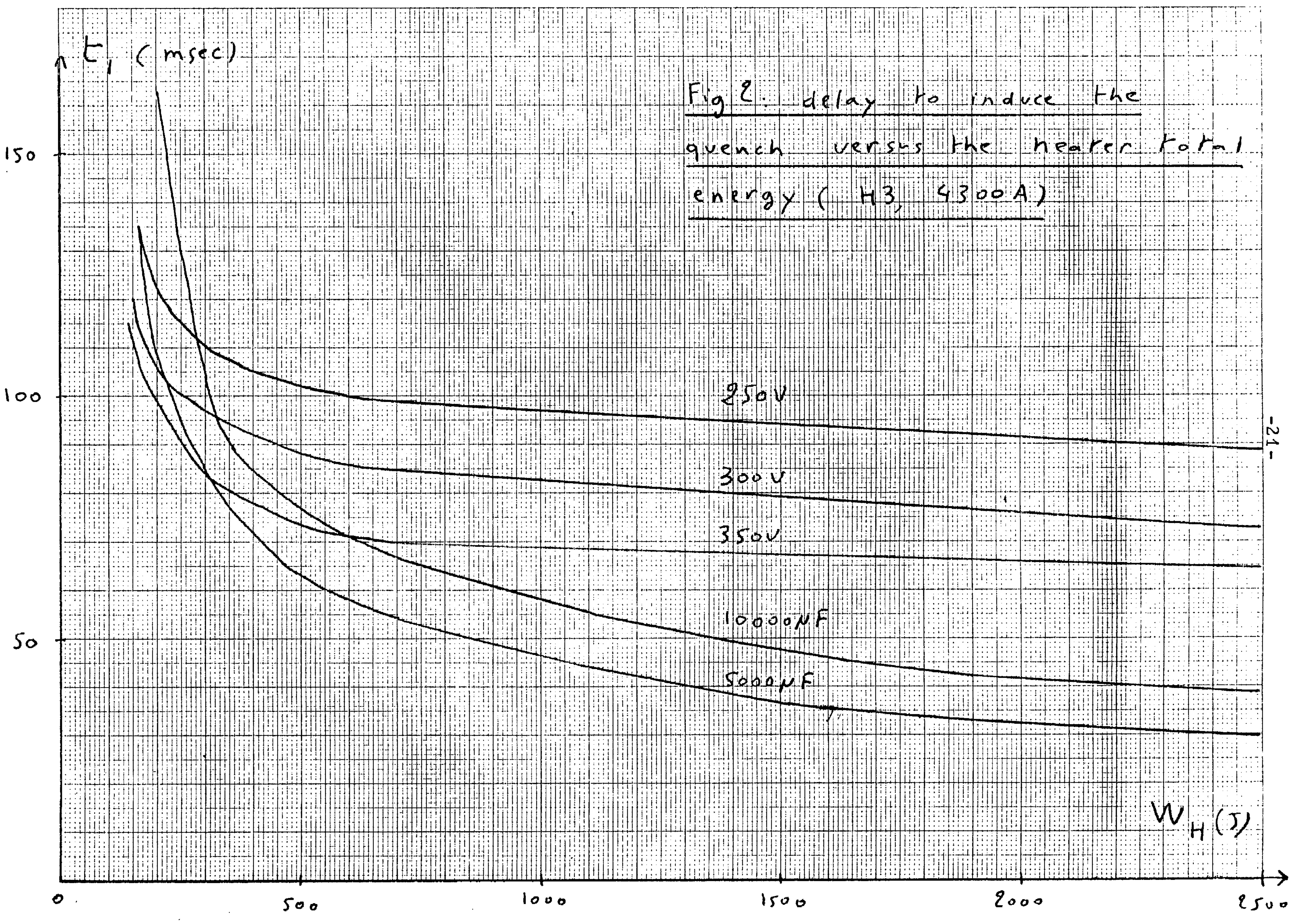

Fig 2. delay to induce the quench versus the heater total energy (H3, 4300A)

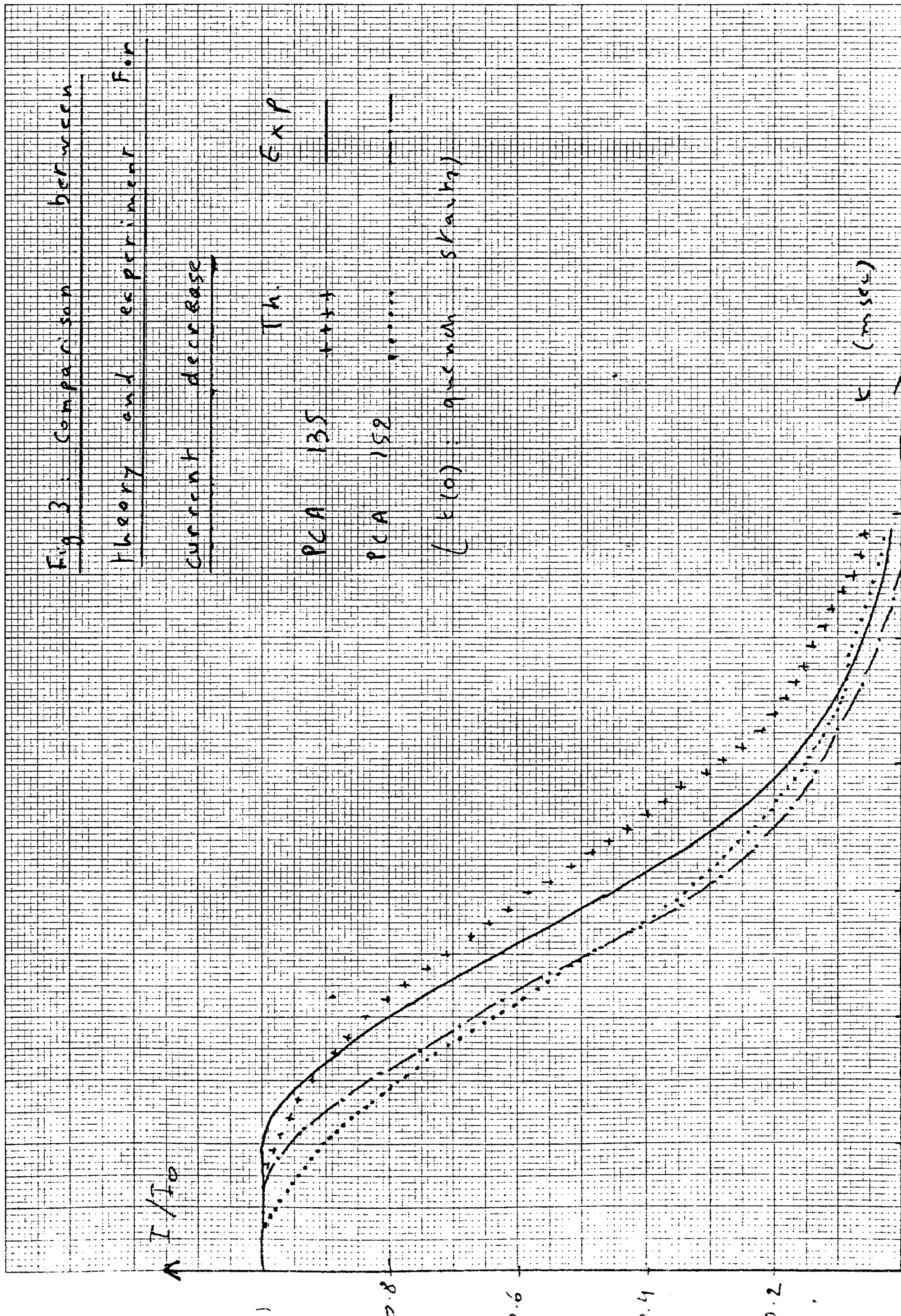
Fig 3. Comparison between
theory and experiment for
current decrease
Th.
Exp
PCA 135
PCA 152
(t(0) quench shown)
t (msec)
I/I0
0
100
200
300
400
500
600
0.2
0.4
0.6
0.8